Activity #6: Circle and Square Investigation (Teacher version) Math

***Note to students*: Teams of two will work to produce a finished product that will answer and explain the problems posed in this activity.**

National Standards addressed:

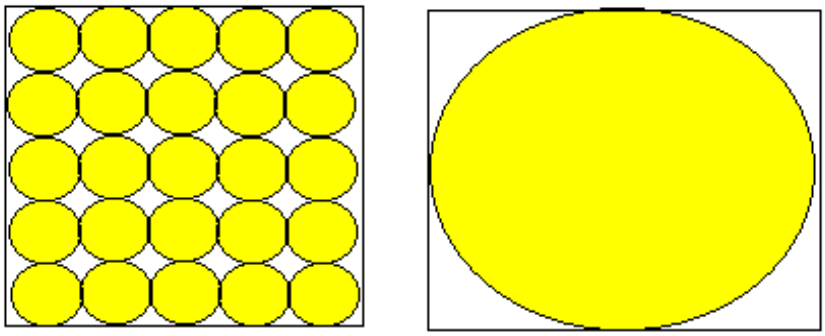
Content Standards:
Algebra Expectations: Students will represent and analyze mathematical situations and structures using algebraic symbols; students will use models to represent and understand quantitative relationships.
Geometry Expectations: Students will explore relationships among classes of geometric objects; make and test conjectures about them, and solve problems involving them; students will use visualization, spatial reasoning and geometric modeling to solve problems.
Measurement Expectation: Students will apply appropriate techniques, tools and formulas to determine measurements.

Process Standards:
Problem Solving Expectations: Students will apply and adapt a variety of appropriate strategies to solve problems; students will monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof Expectations: Students will develop and evaluate mathematical arguments and proofs; students will select and use various types of reasoning and methods of proof.
Communication Expectations: Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others; students will organize and consolidate their mathematical thinking through communication; students will analyze and evaluate the mathematical thinking and strategies of others; students will use the language of mathematics to express mathematical ideas precisely.

***Purpose*:** **To organize mathematical thinking**
To investigate area and circumference
To communicate mathematical reasoning effectively
To develop suitable problem-solving

***Materials*: paper, scissors, stopwatch, compass, ruler, thinking cap, calculator and computer (optional)**

***Activity Explanation*: Given the two patterns below on the same sized squares (Note: Circles and squares are all tangent to one another.), you will investigate and report on the following:**



You are manufacturing Frisbees and will cut circles from squares for pattern templates. You would like to minimize time and waste as part of your strategy to maximize profit. To do this, you will need to answer the following two questions

1. How much more waste will result from cutting the one big circle than from cutting the all the smaller circles? Remember the two squares are the same size.

2. How much longer will it take to cut the all the smaller circles than the one large circle?

(This activity has been modified from Geometry by Prentice Hall ©1998.)

***Activity Procedure*:**

1. You and your partner must plan steps to answer the first question posed. How will you decide on the waste? What will you need to do? Can math help you answer the question? What particular mathematics do you need? Write your plan below. Remember it will be okay to change your plan as you begin to actually follow the steps you propose. It is not necessary to use exactly 6 steps. Use as many as necessary to complete the task.

Step 1: _____

_____

_____

_____

Step 2: _____

_____

_____

_____

Step 3: _____

_____

_____

_____

Step 4: _____

_____

_____

_____

Step 5: _____

Step 6: _____

2. Compare your steps with results from other teams. What do you think about your strategy? Do you want to make any changes? If so, amend your plan. Do not erase the original plan.

3. You are now ready to follow your plan. In journal format, record your findings as you complete each step. Write a paragraph with supportive mathematics stating your answer. Remember that you can change steps at any time. Just be sure to note each change.

4. You and your partner must plan steps to answer the second question posed. How will you decide on the time needed for cutting? What will you need to do? Can math help you answer the question? What particular mathematics do you need? Write your plan below.
Remember it will be okay to change your plan as you begin to actually follow the steps you propose. It is not necessary to use exactly 6 steps. Use as many as necessary to complete the task.

Step 1: _____

Step 2: _____

Step 3: _____

Step 4: _____

Step 5: _____

Step 6: _____

5. Compare your steps with results from other teams. What do you think about your strategy? Do you want to make any changes? If so, amend your plan. Do not erase the original plan.
6. You are now ready to follow your plan. In journal format, record your findings as you complete each step. Write a paragraph, with supportive mathematics, stating your answer.

Analysis and Extension:
Compare your results with results from other teams.

What else would you consider before you went ahead with your Frisbee production?

_____

_____

_____

_____

_____

_____

Many different strategies will surface as you work on this project. Different rubrics and/or performance assessment lists will be needed for different grades. Younger students may use string to measure the circumferences of their models and actually record the time as they cut the circles from their models. Students in a high school geometry or problem solving class will use formulas for area and circumference and may solve using a given square side measurement rather than set the square's side equal to a constant, such as *s*.

Some typical answers might be as follows:

side of square = s units

radius of large circle = $\frac{s}{2}$ units

circumference of large circle = πs units

area of large circle = $\frac{\pi s^2}{4}$ square units

area of square = s^2

radius of small circles = $\frac{s}{10}$ units

circumference of 1 small circle = $\frac{\pi s}{5}$ units

circumference of 25 small circles = $5\pi s$ units

area of 1 small circle = $\frac{\pi s^2}{100}$ square units

area of 25 small circles = $\frac{\pi s^2}{4}$ square units

waste in 25 small circle/square = $s^2 - \frac{\pi s^2}{4}$ square units

waste in 2 large circle/square = $s^2 - \frac{\pi s^2}{4}$ square units

compare circumferences = $\frac{25 \text{ small circles}}{1 \text{ large circle}} = \frac{5\pi s}{\pi s} = \frac{5}{1}$

conclusion to question 1 : same amount of waste

conclusion to question 2 : at least 5 times as long to cut out the 25 small circles

The following web sites and articles provide enrichment and support for this activity:

1. **Geometry by Prentice Hall©1998**
2. http://standards.nctm.org/document/eexamples/chap7/7.3/index.htm#applet
3. http://standards.nctm.org/document/chapter7/geom.htm#bp1
4. http://standards.nctm.org/document/chapter7/conn.htm
5. http://daniel.calpoly.edu/~dfrc/Robin/X-38/X38-index.html